A Parameterized Scheme of Metaheuristics with Exact Methods for determining the Principle of Least Action in Data Envelopment Analysis

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2. Matheuristic
3. Parameterized Scheme of Metaheuristics
4. Hyperheuristic
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Problem Optimization
Main Problem

\[
\begin{align*}
\max \beta_k & - \frac{1}{m} \sum_{i=1}^{m} \frac{t_{ik}^-}{x_{ik}} \\
\text{s.t} \quad & \beta_k + \frac{1}{h} \sum_{r=1}^{h} \frac{t_{rk}^+}{y_{rk}} = 1, \quad (c.1) \\
& - \beta_k x_{ik} + \sum_{j=1}^{n} \alpha_{jk} x_{ij} + t_{ik}^- = 0, \quad \forall i \quad (c.2) \\
& - \beta_k y_{rk} + \sum_{j=1}^{n} \alpha_{jk} y_{rj} - t_{rk}^+ = 0, \quad \forall r \quad (c.3) \\
\sum_{i=1}^{m} v_{ik} x_{ij} + \sum_{r=1}^{h} \mu_{rk} y_{rj} + d_{jk} &= 0, \quad \forall j \quad (c.4) \\
& v_{ik} \geq 1, \quad \forall i \quad (c.5) \\
& \mu_{rk} \geq 1, \quad \forall r \quad (c.6) \\
& d_{jk} \leq Mb_{jk} \quad \forall j \quad (c.7) \\
\alpha_{jk} & \leq M (1 - b_{jk}), \quad \forall j \quad (c.8) \\
& b_{jk} = 0, 1 \quad (c.9) \\
& \beta_k \geq 0 \quad (c.10) \\
& t_{ik}^- \geq 0, \quad \forall i \quad (c.11) \\
& t_{rk}^+ \geq 0, \quad \forall r \quad (c.12) \\
& d_{jk} \geq 0, \quad \forall j \quad (c.13) \\
& \alpha_{jk} \geq 0, \quad \forall j \quad (c.14)
\end{align*}
\]

Inputs \( i = 0 \ldots m; \)
Outputs \( r = 0 \ldots s; \)
DMUs \( j = 0 \ldots N; \)
Problem Decomposition

MIP Problem
Max/Min $c_1 x + c_2 y$
Linear constraints
Integrality constraints
Bound constraints

P1 ($c_1$)
Discrete variables
Metaheuristic

P2 ($c_2$)
Continuous variables
Exact method
Matheuristic
Matheuristic

P1

METAHEURISTIC
IC

1. Generate the discrete variables ($b_{jk}$)

P2

EXACT METHOD

1. Solve the model finding values for the rest of continuous variables (LP problem)
Matheuristic

1. Presolve
2. First generation
3. Improvement
4. Classify solution by fitness
5. Selection
6. Crossover
7. Diversification
8. Improvement

End Condition
Matheuristic

**Metaheuristic**

- **Initialize** → Fix the discrete variable
- **Improve** → Modify the discrete variable
- **Selection** → Fix the reference set
- **Crossover** → Combine solutions
- **Mutation** → Diversify

**Exact method**

- Solve the model
- Fix the continuous variables
Parameterized Scheme
Presolve

Additive model to find the efficient solutions.

Efficient DMUS
A, B, D
Inefficient DMUs
C, E, F
Presolve

All the DMUs to evaluate

Efficient DMUs (set E)

Inefficient DMUs (j-w)
Generation

INEIni (8)  Valid/Invalid  FNEIni (6)

Valid solution
Invalid solution
**Not valid solutions**  
(Transform into valid solution)

- Evaluate a percentage of solutions (PEIIni) and find the not valid constraints
- Try to solve these constraints modifying the affected variables with a local search
- Repeat the process until valid solution or end condition (IIEIni / IIEImp).

**Valid solutions**  
(Improve fitness)

- Modify a single value in a discrete variable in P1 and solve P2 for a percentage of solution (PEIIni).
- If the new value improve the fitness, fix it. If not, discard.
- Repeat the process until end condition (IIEIni / IIEImp).
(*) = Solution selected to be improved
Select solutions to use them in the following iteration:
- Crossover.
- Diversification.

- Solution selected from the valid group.
- Solution selected from the invalid group.

\[ \text{NBESel} = 3 \quad \text{NWESel} = 2 \]
- Just the discrete variables are combined to generate new solutions.
- Inheritance of characteristics
- Crossover by mask.
- Solve the new problem P2 using exact method.

Mask = 101001

PBBCom = 4
PWWCom = 2
Diversification

- **Diversification**: Scape from the local optimum.
- Modify randomly a discrete variable from P1.
- Solve the new P2 problem.
- Improvement included.
End Condition

FNEIni solutions are going to be used for the next iteration.
Hyperheuristics
Hyperheuristic

- A metaheuristic that uses the Matheuristics like solutions, generating and combining them to obtain the best one for each problem.
- Some parameters are included:
  - **HINEIni**: Number of initial Matheuristics.
  - **HP-Com**: Number of combination between the initial population.
- The Hyperheuristic can be trained with a single problem and use the Matheuristic obtained for all the other problems (same model).
- Establish values for all the parameters depending on a specific limit.
Experimental Results
# First Generation

<table>
<thead>
<tr>
<th>size</th>
<th>Hybrid method (10)</th>
<th>Hybrid method (100)</th>
<th>Hybrid method (1000)</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>m n s</td>
<td>% val.</td>
<td>fitness</td>
<td>time(sec)</td>
<td>% val.</td>
</tr>
<tr>
<td>2</td>
<td>55.557.00 0.137  0.01</td>
<td>100.000.00 0.413  6.23</td>
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<tr>
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<td>20.000.00 0.059  0.01</td>
<td>83.925.00 0.355  4.12</td>
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<td>45.4512.72 0.146  2.37</td>
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<tr>
<td>6</td>
<td>20.007.07 0.247  0.02</td>
<td>60.008.36 0.282  3.62</td>
<td>90.009.08 0.398  30.03</td>
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<tr>
<td>7</td>
<td>0.000.00 0.00   0.01</td>
<td>50.0027.38 0.073  1.15</td>
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<table>
<thead>
<tr>
<th>size</th>
<th>Hybrid method (10)</th>
<th>Hybrid method (100)</th>
<th>Hybrid method (1000)</th>
<th>Efficiency</th>
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</thead>
<tbody>
<tr>
<td>m n s</td>
<td>% val.</td>
<td>fitness</td>
<td>time(sec)</td>
<td>% val.</td>
</tr>
<tr>
<td>2</td>
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<td>4</td>
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METAHEURISTIC MODELS

We have established basic metaheuristic models to verify that we can improve these schemes through a hyperheuristic, generating the best metaheuristic.

<table>
<thead>
<tr>
<th>Metaheuristic</th>
<th>IINEIni</th>
<th>FNEIni</th>
<th>PEIIni</th>
<th>IIEIni</th>
<th>NBESel</th>
<th>NWESel</th>
<th>PBBCom</th>
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<td>250</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>100</td>
<td>100</td>
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<tr>
<td>GR</td>
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<td>1</td>
<td>100</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SS</td>
<td>1000</td>
<td>30</td>
<td>50</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>50</td>
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<table>
<thead>
<tr>
<th>Metaheuristic</th>
<th>PWWCom</th>
<th>PEIImp</th>
<th>IIEImp</th>
<th>PEDImp</th>
<th>IDEImp</th>
<th>MNIEnd</th>
<th>NIREnd</th>
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</thead>
<tbody>
<tr>
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<td>100</td>
<td>0</td>
<td>0</td>
<td>10</td>
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<tr>
<td>GR</td>
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<td>0</td>
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<td>0</td>
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<tr>
<td>SS</td>
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<td>50</td>
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Hyperheuristic
### Hyperheuristic

<table>
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<tr>
<th>Problem size</th>
<th>IINEIni</th>
<th>FNEIni</th>
<th>PEIIni</th>
<th>IIEIni</th>
<th>NBESel</th>
<th>NWESel</th>
<th>PBBCCom</th>
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<td>75</td>
<td>11</td>
<td>46</td>
<td>66</td>
<td>15</td>
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<td>50</td>
<td>10</td>
<td>59</td>
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<td>69</td>
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<td>59</td>
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<td>10</td>
<td>68</td>
<td>59</td>
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<td>68</td>
<td>13</td>
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<td>53</td>
<td>63</td>
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<table>
<thead>
<tr>
<th>Problem size</th>
<th>PWWCom</th>
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<th>IIEImp</th>
<th>PEDImp</th>
<th>IIDImp</th>
<th>MNIEnd</th>
<th>NIREnd</th>
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<tbody>
<tr>
<td>$m = 2, n = 50, s = 1$</td>
<td>83</td>
<td>35</td>
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<td>3</td>
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<td>$m = 3, n = 50, s = 2$</td>
<td>100</td>
<td>42</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>12</td>
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<td>4</td>
<td>9</td>
<td>2</td>
<td>10</td>
<td>3</td>
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<tr>
<td>$m = 4, n = 50, s = 3$</td>
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<td>23</td>
<td>7</td>
<td>7</td>
<td>5</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>$m = 5, n = 50, s = 3$</td>
<td>29</td>
<td>33</td>
<td>8</td>
<td>10</td>
<td>8</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>$m = 6, n = 50, s = 4$</td>
<td>41</td>
<td>28</td>
<td>8</td>
<td>9</td>
<td>5</td>
<td>5</td>
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</table>
Hyperheuristic

<table>
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<tr>
<th>Schemes</th>
<th>Hyperheuristic</th>
<th>General Metaheuristic</th>
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<tbody>
<tr>
<td>$m = 2$, $n = 50$, $s = 1$</td>
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<td>0.393197</td>
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<td>0.605534</td>
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<td>0.488011</td>
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<tr>
<td>$m = 6$, $n = 50$, $s = 4$</td>
<td>0.454793</td>
<td>0.429670</td>
</tr>
</tbody>
</table>
Execution Time

- CPLEX
- Hybrid

Execution time (seconds)

Problem size

2-50-1 3-50-2 4-50-2 4-50-3 5-50-3