Scientific Computing and Parallel Programming
Group, University of Murcia
Modelling and optimisation of scientific software in multicore

Domingo Giménez

... and the list of collaborators within the presentation

May 2010, University College Dublin
Contents

1 The Group

2 Scientific code optimisation

3 Modelling basic routines

4 Matrix multiplication
Scientific Computing and Parallel Programming

4 doctors + 5 PhD students, from:
Universidad Miguel Hernández de Elche (2+0)
Centro de Supercomputación de Murcia (0+1)
University of Murcia (2+2)
Universidad Católica de Murcia (0+1)
Universidad Politécnica de Cartagena (0+1)

Information
- Group page: http://www.um.es/pcgum/
- Publications: http://dis.um.es/~domingo/investigacion.html
Research lines

- **Scientific Computing**
  - Mathematical and statistical modelling of scientific problems
  - Development of efficient algorithms to solve these problems
  - Approximated algorithms, metaheuristics
  - Applications of parallelism
- **Parallelism**
  - Execution time modelling
  - Optimization and autooptimization based in the model
  - Application to: algorithms, schemes, scientific problems
  - Adaptation to: multicore, supercomputers, heterogeneous...
- **Applications:**
  - Simultaneous equation models: stat., paral., metah.
  - Computational electromagnetism: paral., metah.
  - Bayesian models: stat., paral.
  - Hydrodynamics: paral.
  - Regional meteorology simulations: paral.
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Regional project

Adaptation and Optimisation of Scientific Code for Hierarchical Computational Systems
joint project with the Computational Electromagnetism Group of the Polytechnic University of Cartagena

- Modelling of parallel scientific codes
- Adaptation of the codes for multicore, supercomputers, heterogeneous systems
- Optimisation and auto-optimisation of the codes
- Applications:
  - Signal filters design
  - Integral equations to study breaking of microstrip components
  - ... others electromagnetic problems
  - Climatic simulations
  - Hydrodynamic
  - Statistics (Simultaneous Equation Models, Bayesian models...)

Spanish project COPABIB

Automatic Building and Optimisation of Parallel Scientific Libraries

[Map showing locations in Spain, including UCD, La Laguna, Castellón, Polit. Valencia, Alicante, Murcia, and others.]
Spanish project COPABIB: research lines

- **Specification of problems, algorithms and architectures**: mathematical formulation and tag-based languages to define specification languages.
- **Software tools for transformation**: translators, symbolic processors and skeletons to obtain libraries from specifications.
- **Matrix algebra libraries**: libraries for dense and sparse linear algebra.
- **Libraries of dynamic programming for optimization problems**: libraries for discrete mathematics problems.
- **Optimization environments**: models, simulators, analyzers, tuning for linear algebra and optimization.
- **Tools for the construction of high-level interfaces**: tools to assist in the construction of interfaces to provide user-friendly access to the libraries.
- **Scientific applications**: interdisciplinary applications using the previous results.
Spanish network

High Performance Computation in Heterogeneous Architectures (CAPAP-H), approximately 25 universities, centres and companies
European network

Open European Network for High Performance Computing on Complex Environments
Numerical Analysis, Libraries and Tools, Mapping, Applications

opportunities for collaboration through the network
Scientific code optimisation

- Modelling scientific code
  - From basic routines...
  - ... to scientific codes
  - For multicore, clusters, supercomputers

- Installation tools and methodology
  - Using the previous models...
  - ... and empirical analysis for the particular routine and computational system

- Adaptation methodology:
  - With the model and the empirical study at installation time...
  - ... adapt the software to the entry and system conditions at running time
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Regional meteorology simulations

Joint work with Sonia Jerez, Juan-Pedro Montávez, Regional Atmospheric Modelling Group, Univ. Murcia

Sonia Jerez, Juan-Pedro Montávez, Domingo Giménez, Optimizing the execution of a parallel meteorology simulation code, IEEE IPDPS, 10th Workshop on Parallel and Distributed Scientific and Engineering Computing, Rome, May 25-29, 2009

- They use MM5, developed at the Pennsylvania State University and the National Centre for Atmospheric Research
- Parallel versions with OpenMP and MPI
- Optimise the use of the parallel codes
- Analysis in multicore systems
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Regional meteorology simulations: modelling

After the simulation of a period of fixed length (*spin-up* period, $T_s$) the influence of the initial condition is discarded. The value of $T_s$ depends on each experiment.

- **Time parallelization:**
  Divide the period $P$ in $N_t$ subperiods and simulate each subperiod with the *spin-up* time $T_s$:

  $T = \left( \frac{P}{N_t} + T_s \right) t$

  where $t$ is the cost of the simulation of a unity-length period

- **Spatial parallelization:** Using the PARALLEL CODE that divides the spatial domain, each portion is solved in a core. Use $N_p = N_x N_y$ cores for each simulation.
  The total number of cores is $N = N_t N_p$
  The cost of a basic operation depends on the parameters: $t = f (N_t, N_x, N_y)$ and mesh configuration.
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Regional meteorology simulations: installation

- A short period of time is simulated for all the possible combinations of $N_t$ with $N_p$
  - with a limit: $N_t N_p \leq 2N$
  - for some trial domains
  - and different mesh shapes: combinations of $N_x$ and $N_y$
- Indicate in the installation:
  - Where package MM5 is
  - The number of available processors
  - Compilation options
  - The manager could decide modify some of the default parameters
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Regional meteorology simulations: execution

- Select at running time the values of $N_t$, $N_x$ and $N_y$
- Taking into consideration the size and characteristics of the problem to be solved
- With the values $t = f(N_t, N_x, N_y)$ estimated at installation time for domains close to the current domain
- To update the information generated at installation time:
  - Overhead
  - Possibly the estimation adjusts better to the problem characteristics
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Regional meteorology simulations: results

- **DEFAUL**: uses default parameters
- **INSTAL**: with installation information selects the values which gives lowest modelled time
- **INS+EXE**: repeats the experiments for the current problem for the parameter combinations which provide lowest modelled time
- **EXECUT**: repeats installation running for the current domain, and selects the parameters which give the lowest estimated time

Reduction between 25% and 40% of the execution time
Hydrodynamic simulations

Joint work with Francisco López-Castejón, Oceanography Group, Polytechnic Univ. of Cartagena


- Study of parallelisation and optimisation of COHERENS (COupled Hydrodynamical-Ecological model for REgioNal and Shelf seas), by the Manag. Unit of the North Sea Math. Models, Napier Univ., Proudman Oceanogr. Lab. and British Oceanogr. Data Centre
- Easy development of parallel multicore versions from existing scientific codes
- Easy optimisation and auto-optimisation methodology
- There are other parallel hydrodynamic codes where this methodology and the previous study could be applied
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Hydrodynamic simulations: modelling

Obtain the execution time of each module, routine and loop in the package

\[
x = \text{Number of nodes in X axe.}
\]
\[
y = \text{Number of nodes in Y axe.}
\]
\[
z = \text{Number of levels in Z axe.}
\]
Hydrodynamic simulations: easy parallelism and optimisation

- Parallelize each loop separately
- with a different number of threads for each loop
- select the number of threads in each loop
  - with information obtained at installation time
  - and adaptation in the initial iterations

Figure 3: Optimum number of cores for a loop with 19 flops, when varying the problem size, in four systems.
Simultaneous Equation Models

Joint work with José-Juan López-Espín, Univ. Miguel Hernández of Elche, Antonio M. Vidal, Polytechnic Univ. Valencia


- Use of matrix decompositions to obtain a number of algorithms with low execution time
- Basic operations: QR decomposition, matrix multiplications, Givens rotations
- Two types of parallelism: in the basic operations, and OpenMP parallelism in the computation of different equations
- Model of the execution time to decide the algorithm to use for an entry and system
- Estimation at installation time of the values of the parameters in the models
- Include two-level parallelism
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Parameterised shared-memory metaheuristics

Joint work with José-Juan López-Espín, Univ. Miguel Hernández of Elche, Francisco Almeida, Univ. of La Laguna

- Parameterised metaheuristic scheme
- facilitates development and tuning of metaheuristics and hybridation/combination of metaheuristics

Initialize(S,ParamInit)
while (not EndCondition(S,ParamEndCond))
    SS = Select(S,ParamSelec)
    if(|SS| >1) SS1 = Combine(SS,ParamComb)
    else SS1 = SS
    SS2 = Improve(SS1,ParamImpr)
    S = Include(SS2,ParamIncl)
Parameterised shared-memory metaheuristics: parallelism

- Unified parallel shared-memory scheme for metaheuristics facilitates development of parallel metaheuristics or of their hybridation/combination
- Parameterised parallel shared-memory scheme for metaheuristics facilitates optimisation of parallel metaheuristics

```c
two-level(MetaheurParam):
omp_set_num_threads(first-level-threads(MetaheurParam))
#pragma omp parallel for
  loop in elements
  second-level(MetaheurParam,first-level-threads)

second-level(MetaheurParam,first-level-threads):
omp_set_num_threads(second-level-threads(MetaheurParam,first-level-threads))
#pragma omp parallel for
  loop in elements
  treat elements
```
Parameterised shared-memory metaheuristics: results

- Applied to obtaining satisfactory Simultaneous Equation Models given a set of values of variables
- Metaheuristics: GRASP, genetic, scatter search, GRASP+genet., GRASP+SS, Gent.+SS, GRASP+genet.+SS
- With different number of threads in each function and two-level parallelism better results

![Graphs showing results for Arabi and Ben](image)
Other scientific problems

- Integral equations to study breaking of microstrip components
  Joint work with José-Ginés Picón, Supercomputing Centre Murcia, and Alejandro Álvarez and Fernando D. Quesada, Computational Electromagnetism Group Univ. Polytechnic of Cartagena
  Parallelise and optimise code, with nested parallelism and basic linear algebra routines (zgemv and zgemm)

- Bayesian simulations
  Joint work with Manuel Quesada, and Asunción Martínez-Mayoral and Javier Socuellamos, Univ. Miguel Hernández
  Web application to study bayesian distributions, to be installed on different platforms and with parallelism hidden to the user

- Possible collaboration with a company: design of bridges, with metaheuristics and parallelism, in supercomputer BenArabi
Modelling basic routines

Joint work with Javier Cuenca, Computer Architecture Department, Univ. of Murcia, Luis-Pedro García, Polytechnic Univ. of Cartagena

- The goal:
  on multicore systems, with OpenMP,
  to model routines of high level
  by using information obtained from routines of low level

- Basic work:
  threads generation
  loop work distribution
  synchronisation

- Higher level routines:
  matrix-vector multiplication
  Jacobi iteration
  matrix-matrix multiplication
  Strassen multiplication
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Modelling basic routines

Joint work with Javier Cuenca, Computer Architecture Department, Univ. of Murcia, Luis-Pedro García, Polytechnic Univ. of Cartagena

- The goal:
  - on multicore systems, with OpenMP,
  - to model routines of high level
  - by using information obtained from routines of low level

- Basic work:
  - threads generation
  - loop work distribution
  - synchronisation

- Higher level routines:
  - matrix-vector multiplication
  - Jacobi iteration
  - matrix-matrix multiplication
  - Strassen multiplication
Modelling: test routines

- **R-generate**
  Creates a series of threads with a fixed quantity of work to do per thread
  To compare the time of creating and managing threads

- **R-pfor**
  A simple for loop where there is a significant work inside each iteration
  To compare the time of distributing dynamically a set of homogeneous tasks

- **R-barriers**
  A barrier primitive set after a parallel working area
  To compare the times to perform a global synchronisation of all the threads
Modelling: systems

- **P2c**
  Intel Pentium, 2.8 GHz, with 2 cores.
  Compilers: icc 10.1 and gcc 4.3.2.

- **A4c**
  Alpha EV68CB, 1 GHz, with 4 cores.
  Compilers: cc 6.3 and gcc 4.3.

- **X4c**
  Intel Xeon, 3 GHz, with 4 cores.
  Compilers: icc 10.1 and gcc 4.2.3.

- **X8c**
  Intel Xeon, 2 GHz, with 8 cores.
  Compilers: icc 10.1 and gcc 3.4.6
Modelling: R-generate

\# threads ≤ \# cores: \( T_{R\_\text{generate}} = PT_{\text{gen}} + NT_{\text{work}} \)

\# threads > \# cores: \( T_{R\_\text{generate}} = PT_{\text{gen}} + NT_{\text{work}} \frac{P}{C} \left( 1 + \frac{T_{\text{swap}}}{T_{\text{cpu}}} \right) \)
Modelling: R-pfor

\[
\text{\# threads } \leq \text{\# cores: } T_{R-pfor} = PT_{gen} + \frac{NT}{P} T_{work} \\
\text{\# threads } > \text{\# cores: } T_{R-pfor} = PT_{gen} + \frac{NT}{C} T_{work} \left( 1 + \frac{T_{swap}}{T_{cpu}} \right)
\]

![Graphs showing performance comparison of different compilers for different scenarios](image)
Modelling: R-barriers

\# \text{threads} \leq \# \text{cores}: \quad T_{R-\text{barriers}} = PT_{\text{gen}} + NT_{\text{work}} + PT_{\text{syn}}

\# \text{threads} > \# \text{cores}: \quad T_{R-\text{barriers}} = PT_{\text{gen}} + NT_{\text{work}} \frac{P}{C} \left(1 + \frac{T_{\text{swap}}}{T_{\text{cpu}}} \right) + PT_{\text{syn}}
Modelling higher routines: Jacobi

- Estimation of the parameters:

<table>
<thead>
<tr>
<th></th>
<th>P2c</th>
<th>X4c</th>
<th>A4c</th>
<th>X8c</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_{gen} (\mu sec)</td>
<td>icc</td>
<td>gcc</td>
<td>icc</td>
<td>gcc</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>25</td>
<td>75</td>
<td>25</td>
</tr>
<tr>
<td>T_{work} (nsec)</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>T_{swap} / T_{cpu}</td>
<td>2</td>
<td>1.5</td>
<td>15</td>
<td>0.8</td>
</tr>
</tbody>
</table>

- Substitution of estimated values of the parameters in the model of the routine:

  - # threads ≤ # cores:

    \[ T_{Jacobi} = PT_{gen} + 11 \frac{n^2}{P} T_{work} \]

  - # threads > # cores:

    \[ T_{Jacobi} = PT_{gen} + 11 \frac{n^2}{C} T_{work} \left( 1 + \frac{T_{swap}}{T_{cpu}} \right) \]

- Decision of the number of threads and compiler to use in the solution of the problem.
Modelling: Jacobi, results

Graphs showing time (seconds) on a logarithmic scale for different numbers of threads.
Modelling: Strassen

# threads ≤ # cores:

\[ T_{Strassen} = P T_{gen} + \frac{7}{4} \frac{n^3}{P} T_{mult} + \frac{9}{2} n^2 T_{add} \]

# threads > # cores:

\[ T_{Strassen} = P T_{gen} + \frac{49}{32} \frac{n^3}{P} T_{mult} \left( 1 + \frac{T_{swap}}{T_{cpu}} \right) + \frac{9}{2} n^2 T_{add} \]

\[ T_{Strassen} = P T_{gen} + \frac{49}{32} \frac{n^3}{C} T_{mult} \left( 1 + \frac{T_{swap}}{T_{cpu}} \right) + \frac{63}{8} \min\{P_1, C\} T_{add} \left( 1 + \frac{T_{swap}}{T_{cpu}} \right) + \frac{9}{2} n^2 T_{add} \]
## Modelling higher routines: Strassen, SP values

<table>
<thead>
<tr>
<th></th>
<th>P2c</th>
<th>X4c</th>
<th>A4c</th>
<th>X8c</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>icc gcc</td>
<td>icc gcc</td>
<td>cc gcc</td>
<td>icc gcc</td>
</tr>
<tr>
<td>$T_{\text{gen}} (\mu\text{sec})$</td>
<td>75 25</td>
<td>75 25</td>
<td>75 25</td>
<td>75 25</td>
</tr>
<tr>
<td>$T_{\text{swap}}/T_{\text{cpu}}$</td>
<td>2+ 0.01P 7- 0.01P</td>
<td>0.9+ 0.3P 0.9+ 0.01P</td>
<td>0.8+ 0.2P 0.8+ 0.02P</td>
<td>6+ 0.05P 0.5+ 0.01P</td>
</tr>
<tr>
<td>$T_{\text{add}} (\mu\text{sec})$</td>
<td>20+ 0.05P 20</td>
<td>23+ 0.3P 30- 0.3P</td>
<td>40+ P 40- 0.1P</td>
<td>10 10</td>
</tr>
<tr>
<td>$T_{\text{mult}} (\rho\text{sec})$</td>
<td>400+ 100P 0.1P</td>
<td>140+ 10P P</td>
<td>60 60- 0.5P</td>
<td>100 100</td>
</tr>
</tbody>
</table>
Modelling: Strassen, results
Modelling: Strassen, results
Modelling higher routines: Strassen, results

Problem size 1000

Combination giving the best results:

<table>
<thead>
<tr>
<th>compiler</th>
<th>P2c</th>
<th>X4c</th>
<th>A4c</th>
<th>X8c</th>
</tr>
</thead>
<tbody>
<tr>
<td>gcc</td>
<td>7</td>
<td>4</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>gcc</td>
<td>7</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Execution time for different values of the parameters:

<table>
<thead>
<tr>
<th></th>
<th>P2c</th>
<th>X4c</th>
<th>A4c</th>
<th>X8c</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCE</td>
<td>1.19</td>
<td>0.50</td>
<td>0.49</td>
<td>0.16</td>
</tr>
<tr>
<td>ORA</td>
<td>1.17</td>
<td>0.49</td>
<td>0.45</td>
<td>0.11</td>
</tr>
<tr>
<td>HW</td>
<td>1.37</td>
<td>0.55</td>
<td>0.65</td>
<td>0.12</td>
</tr>
<tr>
<td>SW</td>
<td>1.22</td>
<td>1.31</td>
<td>1.20</td>
<td>0.32</td>
</tr>
</tbody>
</table>
Matrix multiplication on platforms composed of multicore

The goal:

- To identify the shape matrix multiplication has in a multicore as a function of the problem size and the number of threads, to decide the number of threads to use to obtain the lowest execution time
- To use this information to develop two-level (OpenMP+BLAS) versions of the multiplication, and select the number of threads in each level
- To use this information to develop three-level (MPI+OpenMP+BLAS) versions, and select the number of processes and threads in each level
- To use this information to develop heterogeneous/distributed three-level (MPI+OpenMP+BLAS) versions, and select the number of processes and its distribution or the data partition, and in each processor the number of threads in each level
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Systems, basic components

<table>
<thead>
<tr>
<th>name</th>
<th>architecture</th>
<th>icc</th>
<th>MKL</th>
</tr>
</thead>
<tbody>
<tr>
<td>rosebud05</td>
<td>4 Itanium dual-core 8 cores</td>
<td>11.1</td>
<td>10.2</td>
</tr>
<tr>
<td>rosebud09</td>
<td>1 AMD quad-core 4 cores</td>
<td>11.1</td>
<td>10.2</td>
</tr>
<tr>
<td>hipatia8</td>
<td>2 Xeon E5462 quad-core 8 cores</td>
<td>10.1</td>
<td>10.0</td>
</tr>
<tr>
<td>hipatia16</td>
<td>4 Xeon X7350 quad-core 16 cores</td>
<td>10.1</td>
<td>10.0</td>
</tr>
<tr>
<td>arabi</td>
<td>2 Xeon L5450 quad-core 8 cores</td>
<td>11.1</td>
<td>10.2</td>
</tr>
<tr>
<td>ben</td>
<td>HP Integrity Superdome 128 cores</td>
<td>11.1</td>
<td>10.2</td>
</tr>
</tbody>
</table>
Systems

- **Rosebud (Polytechnic Univ. of Valencia):**
  - 38 cores
  - 2 nodes single-processors, 2 nodes dual-processors, 2 nodes (rosebud05) with 4 dual-core, 2 nodes with 2 dual-core, 2 nodes (rosebud09) with 1 quad-core

- **Hipatia (Polytechnic Univ. of Cartagena):**
  - 152 cores
  - 16 nodes (hipatia8) with 2 quad-core, 2 nodes (hipatia 16) with 4 quad-core, 2 nodes with 2 dual-core

- **BenArabi (Supercomputing Centre of Murcia):**
  - 944 cores
  - Arabi: 102 nodes with 2 quad-core
  - Ben: Hierarchical composition with crossbar interconnection.
  - Two basic components: the computers and two backplane crossbars.
  - Each computer has 4 dual-core Itanium-2 and an ASIC controller to connect the CPUs with the local memory and the crossbar commuters.
  - The maximum memory bandwidth in a computer is 17.1 GB/s and with the crossbar commuters 34.5 GB/s.
  - The access to the memory is non uniform.
  - The user does not control where the threads are assigned.
Using MKL

- The library is multithreaded.
- Number of threads established with the environment variable MKL_NUM_THREADS or in the program with the function mkl_set_num_threads.
- Dynamic parallelism is enabled with MKL_DYNAMIC=true or mkl_set_dynamic(1). The number of threads to use in dgemm is decided by the system, and is less or equal to that established.
- To enforce the utilisation of the number of threads, dynamic parallelism is turned off with MKL_DYNAMIC=false or mkl_set_dynamic(0).
MKL, results

rosebud05

# threads

speed-up

2 3 4 5 6 7 8 9 10 11 12

250
500
750
1000
2000
3000
4000
5000

rosebud09

# threads

speed-up

2 3 4 5 6

250
500
750
1000
2000
3000
4000
5000

hipatia8

# threads

speed-up

2 3 4 5 6 7 8 9 10 11 12

500
1000
4000
8000
MKL, results

Graphs showing speed-up vs. number of threads and cores for benchmarks "arabi" and "ben" with different input sizes: 250, 500, 750, 1000, 2000, 3000, 4000, 5000.
### MKL, results

<table>
<thead>
<tr>
<th>size</th>
<th>Seq.</th>
<th>Max.</th>
<th>Low.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>rosebud05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>0.0081</td>
<td>0.0042</td>
<td>0.0019</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(11)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>rosebud09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>0.0042</td>
<td>0.0050</td>
<td>0.0012</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>hipatia8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>0.0035</td>
<td>0.0021</td>
<td>0.0011</td>
</tr>
<tr>
<td>500</td>
<td>0.026</td>
<td>0.0088</td>
<td>0.0056</td>
</tr>
<tr>
<td>750</td>
<td>0.087</td>
<td>0.021</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(9)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>arabi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>0.0080</td>
<td>0.0015</td>
<td>0.0013</td>
</tr>
<tr>
<td>500</td>
<td>0.034</td>
<td>0.063</td>
<td>0.0049</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(12)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ben</td>
<td></td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>0.021</td>
<td>0.017</td>
<td>0.0014</td>
</tr>
<tr>
<td>500</td>
<td>0.042</td>
<td>0.033</td>
<td>0.0044</td>
</tr>
<tr>
<td>750</td>
<td>0.14</td>
<td>0.063</td>
<td>0.010</td>
</tr>
<tr>
<td>1000</td>
<td>0.32</td>
<td>0.094</td>
<td>0.019</td>
</tr>
<tr>
<td>2000</td>
<td>2.6</td>
<td>0.39</td>
<td>0.12</td>
</tr>
<tr>
<td>3000</td>
<td>8.6</td>
<td>0.82</td>
<td>0.30</td>
</tr>
<tr>
<td>4000</td>
<td>20</td>
<td>1.4</td>
<td>0.59</td>
</tr>
<tr>
<td>5000</td>
<td>40</td>
<td>2.1</td>
<td>1.0</td>
</tr>
</tbody>
</table>
MKL, conclusions

- In rosebud and arabi, the maximum speed-up is achieved for all matrix sizes with more threads than available cores. The dynamic adjustment of threads is a good option, with a thread limit bigger than the number of cores. In hipatia the use of more threads than cores is not advisable. It seems the dynamic selection of threads has been improved in the version 10.2 of MKL.

- In arabi and hipatia, the speed-up changes in stages.

- In Ben to use a large number of cores is not a good option, even with the dynamic adjustment of threads. The optimum number of threads increases with the matrix size, but the speed-up is far from the maximum.
Two-level parallelism

It is possible to use two-level parallelism: OpenMP + MKL. The rows of a matrix are distributed to a set of OpenMP threads ($nthomp$). A number of threads is established for MKL ($nthmkl$). Nested parallelism must be allowed, with OMP_NESTED=true or omp_set_nested(1).

omp_set_nested(1);
omp_set_num_threads(nthomp);
mkl_set_dynamic(0);
mkl_set_num_threads(nthmkl);
#pragma omp parallel
    obtain size and initial position of the submatrix of A to be multiplied
    call dgemm to multiply this submatrix by matrix B
Two-level parallelism, results
Two-level parallelism, results

- **rosebud05**
  - # threads OpenMP vs. # threads MKL
  - Speed-up comparison for different thread counts and MKL configurations.

- **hipatia16**
  - # threads OpenMP vs. # threads MKL
  - Speed-up comparison for different thread counts and MKL configurations.

- **arabi**
  - # threads OpenMP vs. # threads MKL
  - Speed-up comparison for different thread counts and MKL configurations.

- **ben**
  - # threads OpenMP vs. # threads MKL
  - Speed-up comparison for different thread counts and MKL configurations.
Two-level parallelism, conclusions

- In Hipatia (MKL version 10.0) the nested parallelism seems to disable the dynamic selection of threads.
- In the other systems, with dynamic assignation the number of MKL threads seems to be one when more than one OpenMP threads are running.
- When the number of MKL threads is established in the program bigger speed-ups are obtained.
- Normally the use of only one OpenMP thread is preferable.
- Only in Ben to use a higher number of OpenMP threads is a good option. Speed-ups between 1.2 and 1.8 are obtained with 16 OpenMP and 4 MKL threads.
# Two-level parallelism, results

<table>
<thead>
<tr>
<th>size</th>
<th>MKL</th>
<th>2-levels</th>
<th>Sp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>0.0014 (10)</td>
<td>0.0014 (1-10)</td>
<td>1.0</td>
</tr>
<tr>
<td>500</td>
<td>0.0044 (19)</td>
<td>0.0043 (4-11)</td>
<td>1.0</td>
</tr>
<tr>
<td>750</td>
<td>0.010 (22)</td>
<td>0.0095 (4-11)</td>
<td>1.1</td>
</tr>
<tr>
<td>1000</td>
<td>0.019 (27)</td>
<td>0.015 (4-10)</td>
<td>1.3</td>
</tr>
<tr>
<td>2000</td>
<td>0.12 (37)</td>
<td>0.072 (4-16)</td>
<td>1.6</td>
</tr>
<tr>
<td>3000</td>
<td>0.30 (44)</td>
<td>0.18 (4-24)</td>
<td>1.7</td>
</tr>
<tr>
<td>4000</td>
<td>0.59 (50)</td>
<td>0.41 (5-16)</td>
<td>1.4</td>
</tr>
<tr>
<td>5000</td>
<td>1.0 (48)</td>
<td>0.76 (6-20)</td>
<td>1.3</td>
</tr>
<tr>
<td>10000</td>
<td>10 (64)</td>
<td>5.0 (32-4)</td>
<td>2.0</td>
</tr>
<tr>
<td>15000</td>
<td>25 (64)</td>
<td>12 (32-4)</td>
<td>2.1</td>
</tr>
<tr>
<td>20000</td>
<td>65 (64)</td>
<td>22 (16-8)</td>
<td>3.0</td>
</tr>
<tr>
<td>25000</td>
<td>130 (64)</td>
<td>44 (16-8)</td>
<td>3.0</td>
</tr>
</tbody>
</table>
Two-level parallelism, surface shape

Ben, size=3000, reducing the execution time

Execution time with matrix size 3000 in seconds

Only values 1/10 lower than the sequential time

Execution time with matrix size 3000 in seconds

Only values 1/30 lower than the sequential time

Execution time with matrix size 3000 in seconds

Only values 1/40 lower than the sequential time
Two-level parallelism, surface shape

Ben, size=5000, reducing the execution time

Execution time with matrix size 5000, in seconds

- Only times lower than 1/10 the sequential time
- Only times lower than 1/30 the sequential time
- Only times lower than 1/40 the sequential time
Two-level parallelism, surface shape

Execution time with matrix size 5000
only times lower than 1/10 the sequential time
Two-level parallelism, results

Similar results are obtained with other compilers and libraries.

Ben: gcc 4.4 and ATLAS 3.9.
Matrix multiplication: research lines

- Development of a 2IIBLAS prototype, and application to scientific problems
- Simple MPI+OpenMP+MKL version
  Experiments in large shared-memory (ben), large clusters (arabi), and heterogeneous (rosebud)
- ScaLAPACK style MPI+OpenMP+MKL version
  Determine number of processors, and OpenMP and MKL threads
  From the model and empirical analysis or with adaptive algorithm
  In heterogeneous platform the number of processes per processor
- HoHe ScaLAPACK style MPI+OpenMP+MKL version (Vladimir Rychkov?)
  Determine volume of data for each processors, and OpenMP and MKL threads
  From the model and empirical analysis or with adaptive algorithm
- Distributed style MPI+OpenMP+MKL version (Brett Becker?)