Application of metaheuristics to tasks-to-processors assignation problems in heterogeneous systems

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Outline

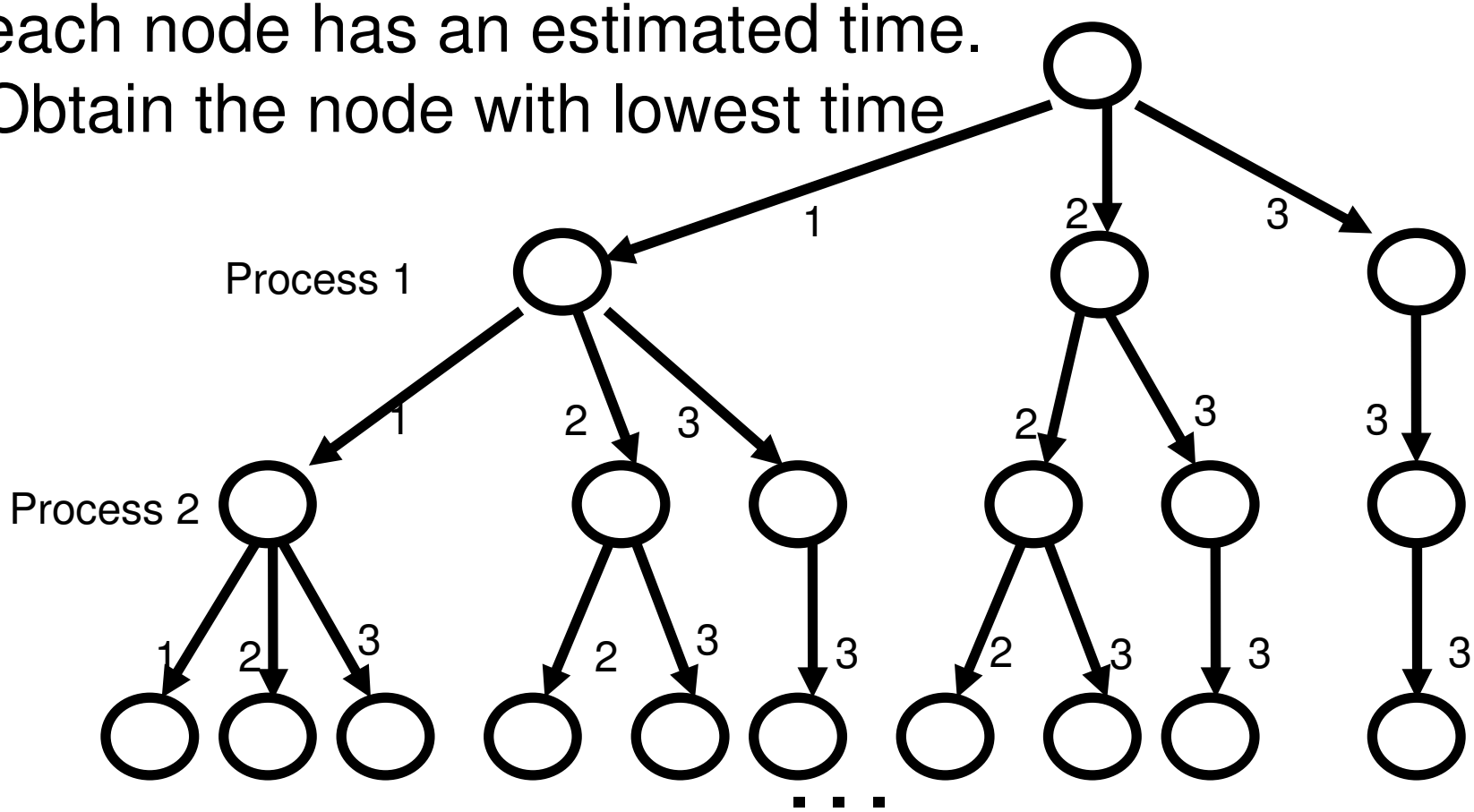
- The problem
- Mapping trees
- Metaheuristic scheme
- A unified class hierarchy for the application of metaheuristics
- A case study
- Conclusions and Future Works

The problem

- Mapping problems. For example:
  - Master-slave scheme
  - Homogeneous processes
- Theoretical model of the execution time.
- A tree of possible mappings. Each node has associated the theoretical cost of the mapping.
- Optimization problem: To obtain a mapping which leads to the lowest total execution time.
- The general case is an NP problem → heuristic methods compulsory.
- A large number of metaheuristics → facilitate development, reuse and experimentation.

Mapping tree I

Tree of processes to processors mappings, each node has an estimated time. Obtain the node with lowest time



Mapping tree I: Execution time model

- Heterogeneous computation: t_{ci}
- Heterogeneous communication: t_{sij}, t_{wij}
- Execution time:

$$t = t_c\, t_{comp} + t_s\, t_{start} + t_w\, t_{word}$$

$$t_c = \max\{d_i\, t_{ci}\}$$

$$t_s = \max\{d_i \neq 0,\ d_j \neq 0 : t_{sij}\}$$

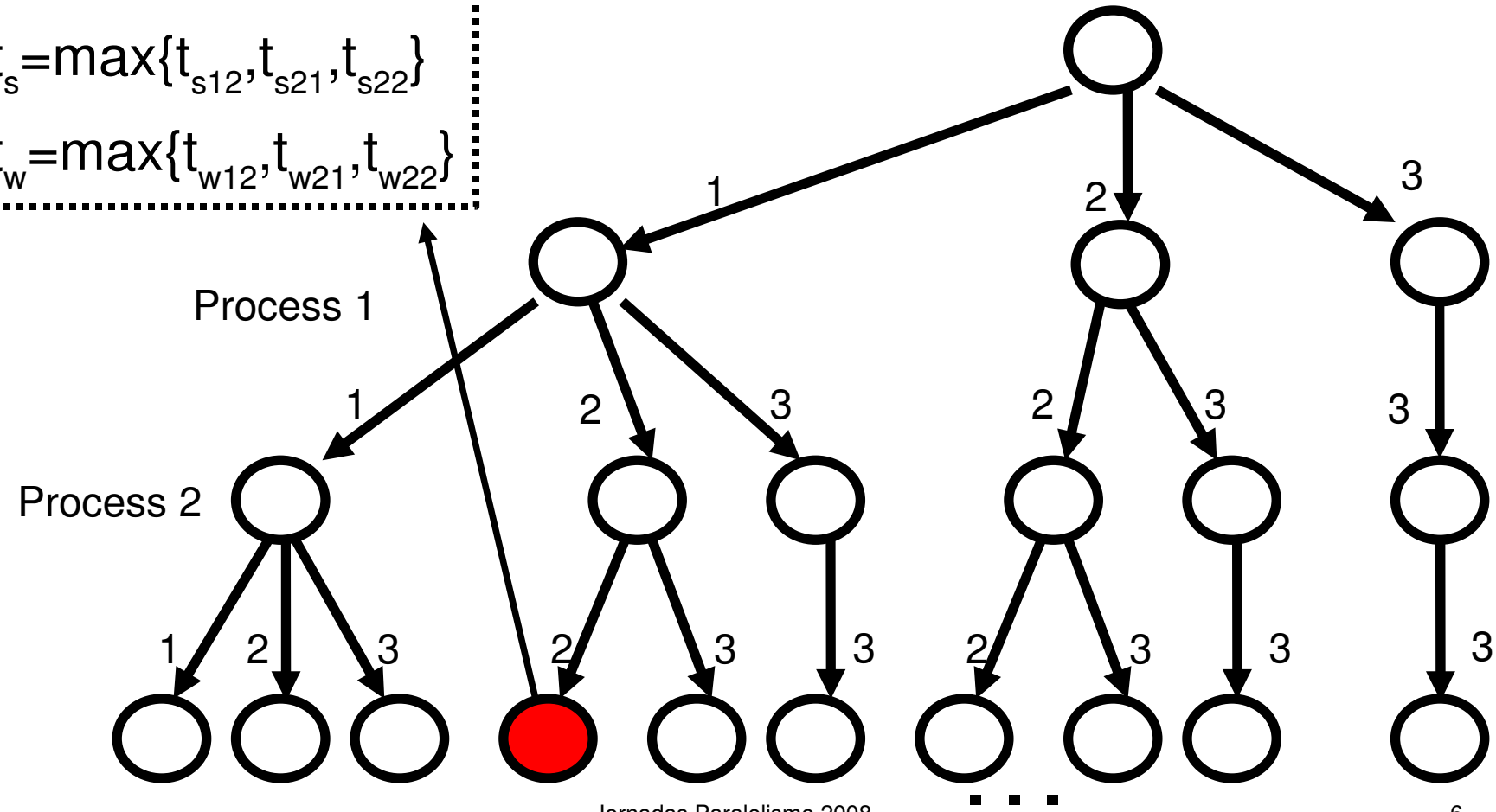
$$t_w = \max\{d_i \neq 0,\ d_j \neq 0 : t_{wij}\}$$

Mapping tree I: Execution time model

$t_c = \max\{t_{c1}, 2t_{c2}\}$

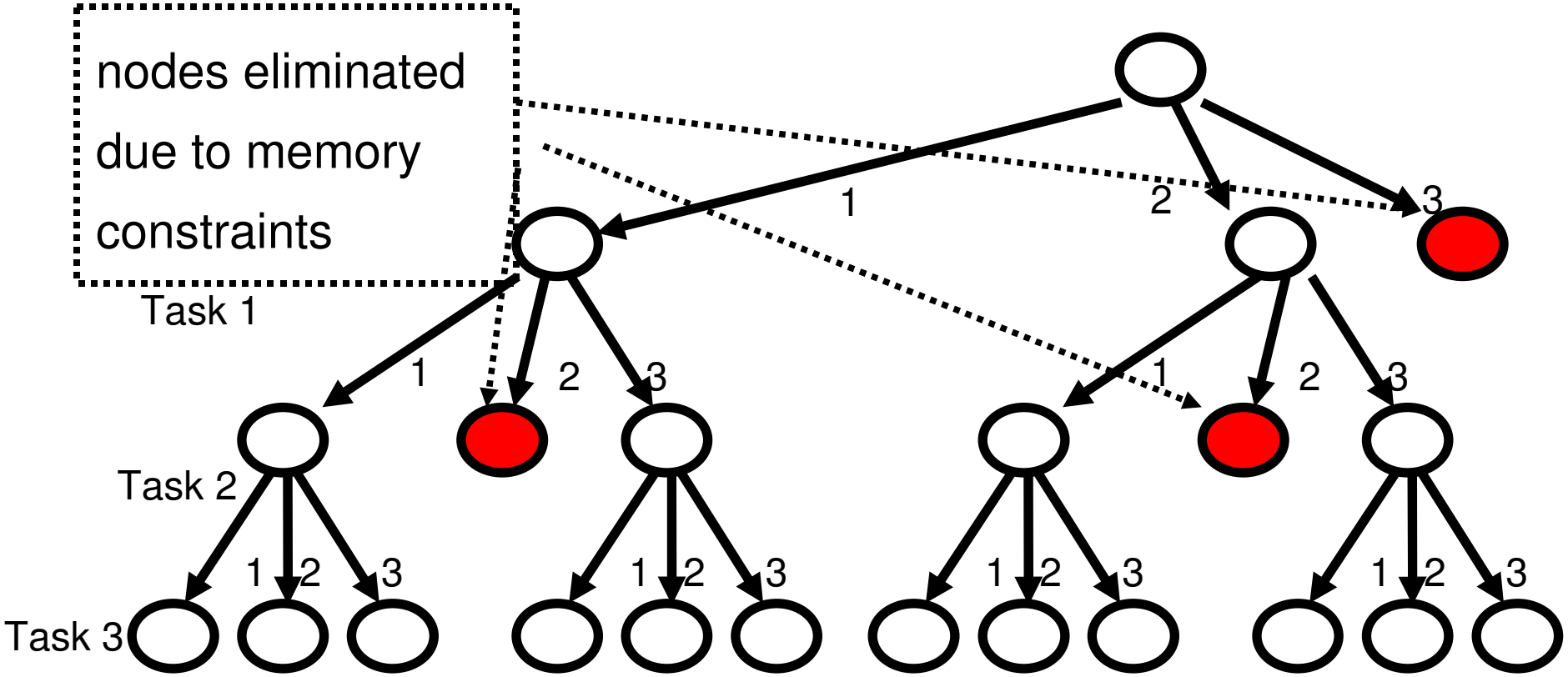
$t_s = \max\{t_{s12}, t_{s21}, t_{s22}\}$

$t_w = \max\{t_{w12}, t_{w21}, t_{w22}\}$



Mapping tree II

A set of tasks generated by a master processor, with memory constraints



Mapping tree II: Execution time model

- Heterogeneous computation: t_{ci}
- No communications
- Execution time of each node (mapping d):

$$\max_{j=1,\ldots,P} \{t_{cj} \sum_{l=1,\ldots,T,\ d_l=j} c_l\}$$

- Optimization problem:

$$\min_d \max_{j=1,\ldots,P} \{t_{cj} \sum_{l=1,\ldots,T,\ d_l=j} c_l\}$$

Metaheuristic scheme

```
Initialize(C)
WHILE (NOT EndCondition(C))
    S = ObtainSubset(C)
    IF |S|>1
        S1= Combine(S)
    ELSE
        S1 = S
    ENDIF
    S2 = Improve(S1)
    C = IncludeSolutions(S2)
ENDWHILE
```

Substitute routines for the particular metaheuristic

→

class hierarchy

Metaheuristic scheme

```
Initialize(C)
WHILE (NOT EndCondition(C))
    S = ObtainSubset(C)
    IF |S|>1
        S1= C
    ELSE
        S1 = S
    ENDIF
    S2 = Impro
    C = Includ
ENDWHILE
```

Initialize. To create each individual of the initial set *S*. Assign tasks to processors with a probability proportional to the processor speed

- **GA**: a large initial population of assignations
- **SS**: a reduced number of elements in *S*
- **TS:** a set *S* with only one element
- **GR**: a set is initially generated, or one element could be generated in each iteration

Metaheuristic scheme

```
Initialize(C)
WHILE (NOT E
    S = ObtainS
    IF |S|>1
        S1= Co
    ELSE
        S1 = S
    ENDIF
    S2 = Improv
    C = Include
ENDWHILE
```

ObtainSubset: Some of the individuals are selected randomly.

- **GA**: Pairs of individuals are selected.
- **SS**: It is possible to select all the elements for combination, or to select the best elements to be combined with the worst ones.
- **TS**: This function is not necessary because $|S| = 1$.
- **GR**: One element from the set of solutions is selected to constitute the set *SS* ($|SS| = 1$). More probability is assigned to mappings with low cost

Metaheuristic scheme

```
Initialize(C)
WHILE (NOT
    S = Obtai
    IF |S|>1
        S1= C
    ELSE
        S1 = S
    ENDIF
    S2 = Improve(S1)
    C = IncludeSolutions(S2)
ENDWHILE
```

Combine: The selected individuals are crossed, and *SS*1 is obtained.

- **GA**, **SS**: The individuals can be crossed in different ways.
- **TS**, **GR**: This function is not necessary.

Metaheuristic scheme

```
Initialize(C)
WHILE (NOT E
    S = Obtain
    IF |S|>1
        S1= Co
    ELSE
        S1 = S
    ENDIF
    S2 = Impro
    C = Include
ENDWHILE
```

Improve:

- **GA**: A few individuals are selected to obtain other individuals, which can differ greatly (mutation operands).
- **SS**: A greedy method:
  - Take a task assigned to the most loaded processor.
  - Reassign it to another processor.
  - Repeat while the cost continues to be reduced.
- **TS**: Some elements in the neighborhood are analysed, excluding tabu elements.
- **GR**: The greedy method of **SS** is applied.

Metaheuristic scheme

```
Initialize(C)
WHILE (NOT E
    S = Obtain
    IF |S|>1
        S1= Co
    ELSE
        S1 = S
    ENDIF
    S2 = Improve(S1)
    C = IncludeSolutions(S2)
ENDWHILE
```

IncludeSolutions: Selects some elements of *SS*2 to be included in *S* for the next iteration.

GA: The best individuals from the original set, their descendants and the individuals obtained by mutation.

SS: The best elements are selected, as well as some scattered elements, to avoid falling within local minima.

TS, **GR**: The best element from those analysed is taken as the next solution.

Metaheuristic scheme

```
Initialize(C)
WHILE (NOT EndCondition(C))
    S = ObtainSubset(C)
    IF |S|>1
        S1= Co
    ELSE
        S1 = S
    ENDIF
    S2 = Improve(S1)
    C = IncludeSolutions(S2)
ENDWHILE
```

EndCondition:

GA, **SS**, **TS**, **GR**: maximum number of iterations, or that the best fitness value does not change over a number of iterations.

Class hierarchy

- Reuse classes and methods
- Develop new metaheuristics
- Tune parameters and functions for the problem
- Obtain a satisfactory metaheuristic for the problem

A case study: problem II

Mapping time and modelled execution time (in seconds), varying the number of tasks

	GA		SS		TS		GR	
tasks	map.	simul.	map.	simul.	map.	simul.	map.	simul.
25	0.139	1484	0.259	**1450**	0.010	**1450**	0.045	**1450**
50	0.413	1566	0.429	1900	0.015	1757	0.078	**1524**
100	0.592	1903	0.834	1961	0.022	3018	0.158	**1460**
200	0.825	**3452**	1.540	**3452**	0.079	**3452**	0.293	**3452**
400	3.203	**3069**	2.682	3910	0.375	**3069**	0.698	**3069**

A case study: problem II

PCGUM

Comparison of the different tunings applied to the Genetic Algorithm, varying the number of tasks

	basic GA		T1		T2		T3		T4		T2+T3		T2+T4		T3+T4	
tasks	map.	simul.	map.	simul.	map.	simul.	map.	simul.	map.	simul.	map.	simul.	map.	simul.	map.	simul.
50	0.13	1646	0.02	2277	0.05	**1524**	0.08	1715	0.09	1715	0.05	**1524**	0.06	**1524**	0.08	1715
100	0.25	2068	0.09	2581	0.13	**1460**	0.14	2230	0.25	2000	0.17	**1460**	0.16	**1460**	0.14	2230
150	0.47	2422	0.19	2908	0.19	**2039**	0.25	2464	0.36	2418	0.22	**2039**	0.22	**2039**	0.25	2464
200	0.41	**3452**	0.28	3717	0.31	**3452**	0.31	**3452**	0.33	**3452**	0.34	**3452**	0.34	**3452**	0.33	**3452**
400	1.56	**3069**	1.19	4184	1.19	**3069**	1.67	**3069**	1.42	**3069**	1.20	**3069**	1.25	**3069**	1.72	**3069**
1600	12.10	3680	10.50	4061	11.77	**1735**	11.38	3882	12.08	3482	12.56	**1735**	11.28	**1735**	12.09	3882

Conclusion, future works

- Hierarchy of metaheuristics
- Easy development, tuning and experimentation

- Application to other mapping problems
- Class hierarchy for general mapping problems
- Reuse among different problems

Thanks

Questions?

Comments?

Criticisms?

Application of Metaheuristics to the Scheduling Problem:

Advanced Tuning of the Genetic Algorithm

PCGUM

- In **Combine**: to change the heredity method:
  - **T1**: Each component is inherited pseudo-randomly, giving more probability to the parent with better fitness value.
  - **T2**: Choosing each component of a descendant from the less loaded processor of those of its parents.
    - The load of a processor *r* : $$W_r = a_r \sum_{\{l=0,1,\ldots,t-1;d_l=r\}} c_l$$

Application of Metaheuristics to the Scheduling Problem:

Advance Tuning of the Genetic Algorithm

PCGUM

- **T3**. In **Improve:** a hybrid approach, using a steered mutation:
  - Each task assigned to an overloaded processor is randomly reassigned.

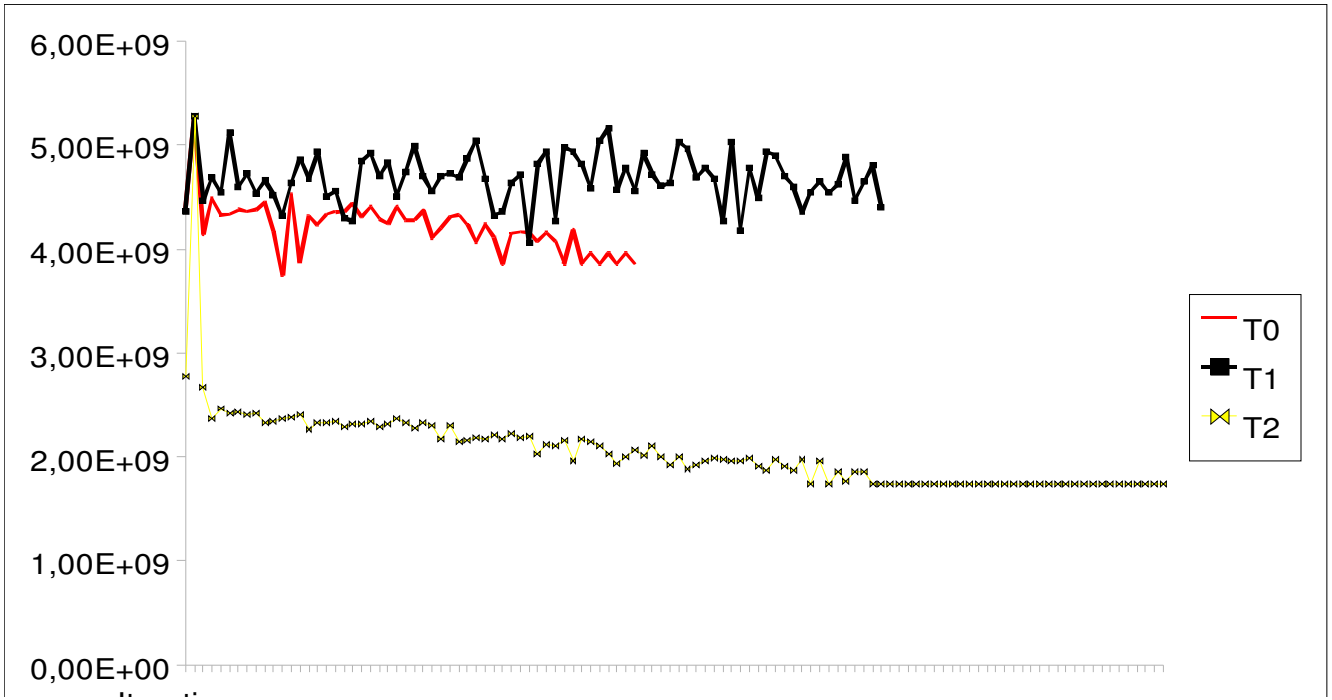
  →The goal of this mutation is to reduce the total load of the most overloaded processors.

- **T4**. In **ObtainSubset:**
  - To chose pseudo-randomly the solutions that will be combined, giving more probability to the solutions with better fitness.

Application of Metaheuristics to the Scheduling Problem:

Advance Tuning of the Genetic Algorithm

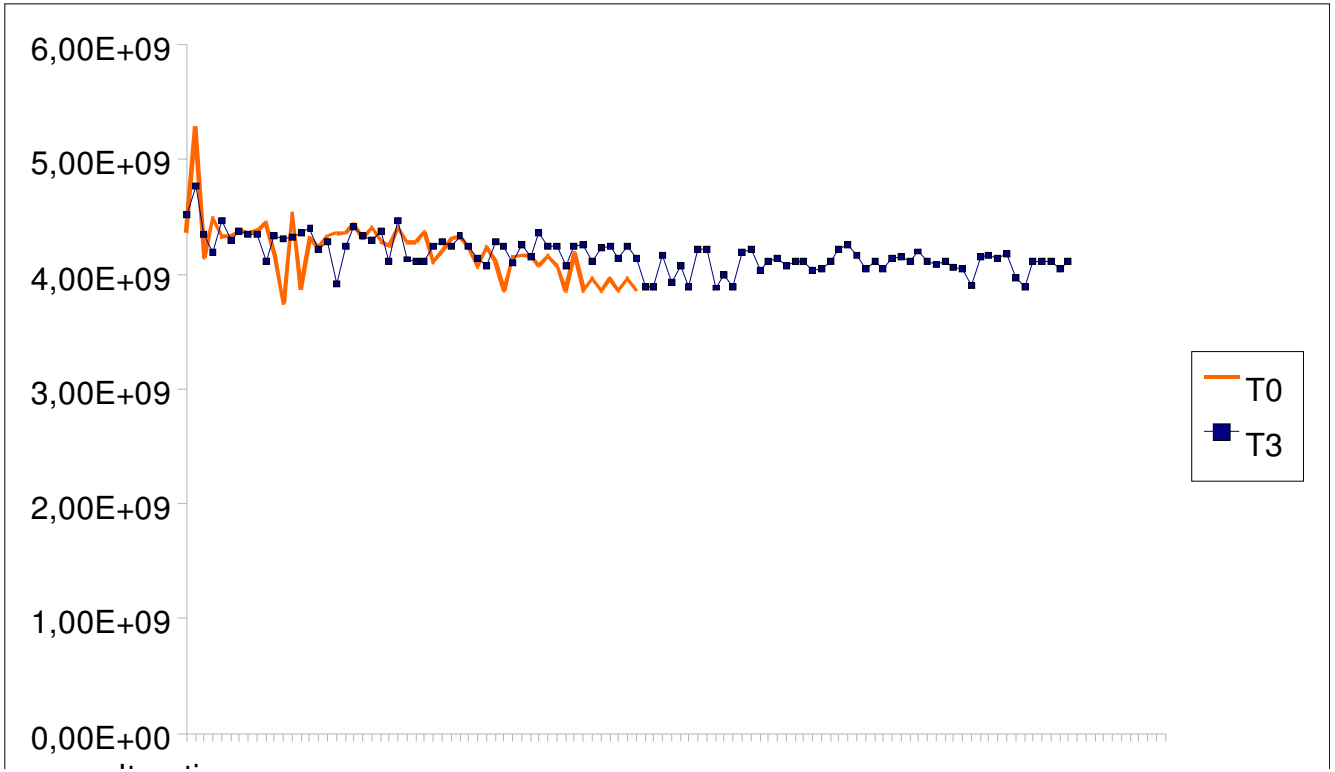
Evolution of the best solution from the generated individuals per iteration, for a problem size of 1600 tasks, without tuning (T0) applied to routine **Combine**, with T1 and with T2



Application of Metaheuristics to the Scheduling Problem:

Advance Tuning of the Genetic Algorithm

Evolution of the best solution from the generated individuals per iteration, for a problem size of 1600 tasks, without tuning (T0) applied to routine **Improve**, and with T3



Application of Metaheuristics to the Scheduling Problem:

Advance Tuning of the Genetic Algorithm

Evolution of the best solution from the generated individuals per iteration, for a problem size of 1600 tasks, without tuning (T0) applied to routine **ObtainSubset**, and with T4